Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2018

July 29, 2019

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2018	(2017)	(2016)	(2015)	(2014)	2018	(2017)	(2016)	(2015)	(2014)
I	58	(52)	(56)	(48)	(49)	38.16	(39.39)	(39.72)	(32.88)	(31.01)
II.1	67	(64)	(58)	(69)	(78)	44.08	(48.48)	(41.13)	(47.26)	(49.37)
II.2	25	(11)	(24)	(25)	(21)	16.45	(8.33)	(17.02)	(17.12)	(13.29)
III	2	(3)	(3)	(3)	(9)	1.32	(2.27)	(2.13)	(2.05)	(5.7)
P	0	(2)	(0)	(1)	(1)	0	(1.52)	(0)	(0.68)	(0.63)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	152	(132)	(141)	(146)	(158)	100	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

BE Extended Essays, BSP projects, and coursework submitted for the History of Mathematics course, the Mathematics Education course and the Undergraduate Ambassadors Scheme, were double marked.

The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 5 on page 12.

B. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

C. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 6 February 2018 and the second notice on 1 May 2018.

All notices and the examination conventions for 2018 are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chairman would particularly like to thank Gemma Proctor for administering the whole process with efficiency, and also to thank Nia Roderick, Charlotte Turner-Smith and Waldemar Schlackow.

In addition the internal examiners would like to express their gratitude to Professor Blackburn and Professor Branicki for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2017 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2017 Part A examination, in which the 2018 Part B cohort were awarded their USMs for Part A;
- a document issued by the Mathematics Teaching Committee giving broad guidelines on the proportion of candidates that might be expected in each class, based on the class percentages over the last five years in Mathematics Part B, Mathematics & Statistics Part B, and across the MPLS Division.

Having said this, as in Table 1 the proportion of first class degrees in Mathematics alone awarded (38.64%) was high, and the proportion of II.2 and below degrees in Mathematics awarded (12.88%) was low, compared to the guidelines. One reason for this is that the examiners consider candidates in Mathematics and in Mathematics and Statistics together when determining USMs, and this year the Mathematics and Statistics candidates performed poorly compared to the Mathematics candidates, so that the averages for the two schools combined (35.2% firsts, and 14.2% II.2 and below) are consistent with the Teaching Committee guidelines.

It seems plausible that the increase in time this year from 1.5 hours to 1.75 hours for Mathematics unit papers may have helped candidates near the II.1/II.2 borderline to perform better, leading to fewer II.2s. The number of candidates was also low (132, compared to an average of 155 over 2008-2016), which may have been in part due to withdrawals by candidates with problems likely to lower their performance, raising the overall standard.

Setting and checking of papers and marks processing

Requests to course lecturers to act as assessors, and to act as checkers of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions. The questions were initially set by the course lecturer, in almost all cases with the lecturer of another course involved as checkers before the first drafts of the questions were presented to the examiners. Most assessors acted properly, but a few failed to meet the stipulated deadlines (mainly for Michaelmas Term courses) and/or to follow carefully the instructions provided.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses which had been submitted in time; consideration of the remaining papers had to be deferred. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule.

Camera ready copy of each paper was signed off by the assessor, and then submitted to the Examination Schools.

Except by special arrangement, examination scripts were delivered to the Mathematical Institute by the Examination Schools, and markers collected their scripts from the Mathematical Institute. Marking, marks processing and checking were carried out according to well-established procedures. Assessors had a short time period to return the marks on standardised mark sheets. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the mark sheets.

All scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers under the supervision of Helen Lowe sorted all the scripts for each paper for which the Mathematics Part B examiners have sole responsibility, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way, errors were corrected with each change independently verified and signed off by one of the examiners, who were present throughout the process. A small number of errors were found, but they were mostly very minor and hardly any queries had to be referred to the marker for resolution.

Throughout the examination process, candidates are treated anonymously, identified only by a randomly-assigned candidate number, until after all decisions on USMs, degree classes, Factors Affecting Performance applications, prizes, and so on, have been finalized.

This year, there were a few more corrections to papers announced during the examinations than usual (of 31 papers, 4 papers had one correction, and 4 papers had two separate corrections). There appears to be no pattern on MT/HT or Pure/Applied papers receiving corrections. This may have been a failure of vigilance on the part of the board of examiners, but we also feel that not all of our colleagues put as much effort as they should (and a few, very little effort) into proofreading their draft papers.

Standard and style of papers

At the beginning of the year all setters were asked to aim that a I/II.1 borderline candidate should get about 36 marks out of 50, and that a II.1/II.2 borderline script should get about 25 marks, and emphasising the problems caused by very high marks.

This year one paper (B5.3) turned out to be too easy. This causes problems with determining USMs at the top end.

Setting papers that are significantly too easy (and marking such papers generously) is undesirable from the point of view of fairness. Such papers generate more USMs than usual in the range 80-100 from candidates with close to full marks. An undergraduate who has the good fortune to take an easy paper and score highly will typically receive a rather higher USM than he or she would otherwise have done – perhaps a USM of 100 – and this can easily push an otherwise high II.1 candidate into the first class.

Timetable

Examinations began on Monday 28 May and finished on Friday 15 June.

Determination of University Standardised Marks

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part A performances of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part A average USMs in the ranges $[69.5, 100]$, $[59.5, 69.5)$ and $[0, 59.5)$, respectively.

The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the number of I and II.1 candidates in Part A, as given by N_1 and N_2, is the same as the I and II.1 number of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs, and the Examiners may then adjust them to take account of consultations with assessors (see above) and their own judgement. The examiners have scope to make changes, either globally by changing certain parameters, or on individual papers usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm. They also have the option to introduce additional corners. For a well-set paper taken by a large number of candidates, the algorithm yields a piecewise linear map which is fairly close to linear, usually with somewhat steeper first and last segments. If the paper is too easy or too difficult, or is taken by only a few candidates, then the algorithm can yield anomalous results—very steep first or last sections, for instance, so that a small difference in raw mark can lead to a relatively large difference in USMs. For papers with small numbers of candidates, moderation may be carried out by hand rather than by applying the algorithm.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held ahead of the first plenary examiners' meeting to assess the results produced by the algorithm, to identify problematic papers and to try some experimental changes to the scaling of individual papers. This provided a starting point for the first plenary meeting to obtain a set of USM maps yielding a tentative class list with class percentages roughly in line with historic data.

The first plenary examiners' meeting, jointly with Mathematics & Statistics examiners, began with a brief overview of the methodology and of this year's data. Then we considered the scaling of each paper, making provisional adjustments in some cases. The full session was then adjourned to allow the examiners to look at scripts. This was both to help the external examiners to form a view of overall standards, and to answer questions that had arisen on how best to scale individual papers; for instance, to decide whether a given raw mark should correspond to the I/II.1 or II.1/II.2 borderline, an examiner would read all scripts scoring close to this raw mark, and make a judgement on their standard.

The examiners reconvened and we then carried out a further scrutiny of the scaling of each paper, making small adjustments in some cases before confirming the scaling map (those Mathematics & Statistics examiners who were not Mathematics examiners left the meeting once all papers with significant numbers of Mathematics & Statistics candidates had been considered).

Table 2 on page 9 gives the final positions of the corners of the piecewise linear maps used to determine USMs.

The Mathematics examiners reviewed the positions of all borderlines for their cohort. For candidates very close to the proposed borderlines, marks profiles and particular scripts were reviewed before the class list was finalised.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Factors affecting performance

A subset of the examiners had a preliminary meeting to consider the submissions for factors affecting performance in Part B. There were nine Part 13 submissions which the preliminary meeting classified in bands 1, 2, 3 as appropriate. The full board of examiners considered the nine cases in the final meeting, and the certificates passed on by the examiners in Part A 2017 were also considered. All candidates with certain conditions (such as dyslexia, dyspraxia, etc.) were given special consideration in the conditions and/or time allowed for their papers, as agreed by the Proctors. Each such paper was clearly labelled to assist the assessors and examiners in awarding fair marks. Details of cases in which special consideration was required are given in Section E.2.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	Additional Corners	N_1	N_2	N_3
B1.1	(17.40, 37)	(30.3, 57)	(41.8, 72)		12	21	8
B1.2	(13.61, 37)	(23.7, 57)	(40.2, 72)		24	27	13
B2.1	(10.39, 37)	(20, 57)	(37, 72)		21	14	1
B2.2	(11.25, 37)	(19.6, 57)	(33, 72)		25	14	4
B3.1	(11.89, 37)	(20.7, 57)	(37.2, 72)		25	16	4
B3.2	(11, 37)	(27, 57)	(38.5, 72)		10	3	2
B3.3	(13.9, 37)	(26, 57)	(40, 72)		13	7	1
B3.4	(13, 37)	(23, 57)	(37, 72)		22	14	4
B3.5	(16, 37)	(32.5, 57)	(40, 72)		18	11	5
B4.1	(10.34, 37)	(23, 57)	(32, 72)		18	13	2
B4.2	(12.06, 37)	(21, 57)	(36, 72)		17	10	2
B4.3	(0, 0)	(50, 100)			1	1	0
B5.1	(14.53, 37)	(25.3, 57)	(38.8, 72)		12	24	12
B5.2	(13, 37)	(25, 57)	(40, 72)		19	26	14
B5.3	(11.66, 37)	(20.3, 57)	(33.8, 72)		11	19	7
B5.4	(11.89, 37)	(20.7, 57)	(37.2, 72)		9	18	6
B5.5	(15, 37)	(27.3, 57)	(40.8, 72)		8	25	10
B5.6	(12, 37)	(23, 57)	(36.4, 72)		13	17	9
B6.1	(15, 41)	(24, 57)	(41, 70)		6	15	10
B6.2	(18.95, 37)	(33, 57)	(44, 72)		3	8	6
B6.3	(10.85, 37)	(17, 57)	(28, 72)		5	8	6
B7.1	(15.33, 37)	(26.7, 57)	(40, 72)		4	11	5
B7.2	(11.48, 37)	(20, 57)	(31.5, 72)		2	8	0
B7.3	(13.95, 37)	(24.3, 57)	(37.8, 72)		5	14	3
B8.1	(14.53, 37)	(25.3, 57)	(38.8, 72)		25	12	11
B8.2	(13.78, 37)	(24, 57)	(39, 72)		19	7	5
B8.3	(8, 37)	(24, 57)	(41, 72)		18	37	20
B8.4	(17.06, 37)	(24.5, 57)	(42, 72)		0	11	5
B8.5	(12, 37)	(28, 57)	(40.4, 72)		8	16	10
SB1	(17.92, 37)	(31.2,57)	(55.2, 72)		7	18	6
SB2a	(12.23, 37)	(21.3, 57)	(33.5, 72)		11	18	5
SB2b	(8, 37)	(24, 57)	(42, 70)		10	28	10
SB3a	(12, 37)	(24.6, 57)	(36.6, 72)		33	48	17
SB3b	(10.97, 37)	(18.5, 57)	(35.5, 70)		5	14	3
SB4a	(15.91, 37)	(27, 57)	(42, 72)		3	21	11
SB4b	(14.07, 37)	(24.5, 57)	(41, 72)		2	13	6

Table 3 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 3: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
89	1	1	0.66
88	2	2	1.32
87	3	3	1.97
86	4	4	2.63
83	5	6	3.95
82	7	9	5.92
81	10	11	7.24
79	12	14	9.21
78	15	16	10.53
77	17	18	11.84
76	19	24	15.79
75	25	31	20.39
74	32	38	25
73	39	42	27.63
72	43	46	30.26
71	47	57	37.5
70	58	58	38.16
69	59	62	40.79
68	63	72	47.37
67	73	78	51.32
66	79	84	55.26
65	85	96	63.16
64	97	101	66.45
63	102	107	70.39
62	108	111	73.03
61	112	121	79.61
60	122	125	82.24
59	126	132	86.84
58	133	134	88.16
57	135	137	90.13
56	138	143	94.08
55	144	146	96.05
54	147	147	96.71
53	148	149	98.03
50	150	150	98.68
47	151	151	99.34
42	152	152	100

B. Equality and Diversity issues and breakdown of the results by gender

Table 4: Breakdown of results by gender

Class	Number								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	9	49	58	7	45	52	10	46	56
II.1	15	52	67	21	43	64	17	41	58
II.2	9	16	25	5	6	12	10	14	24
III	0	2	2	0	3	3	2	1	3
P	0	0	0	0	2	2	0	0	0
Total	33	119	152	33	99	133	39	102	141
Class	Percentage								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	27.27	41.18	38.16	21.21	45.45	39.39	25.64	45.1	39.72
II.1	45.45	43.7	44.08	63.63	43.43	48.48	43.59	40.32	41.13
II.2	27.27	13.45	16.45	15.15	6.06	8.33	25.64	13.73	17.02
III	0	1.68	1.32	0	3.03	2.27	5.13	0.98	2.13
P	0	0	0	0	2.02	1.52	0	0	0
Total	100	100	100	100	100	100	100	100	100

Table 4 shows the performances of candidates broken down by gender. The examiners were concerned to discover, after the class lists were agreed, that the percentage of male candidates awarded first class degrees was over double the percentage of female candidates awarded first class degrees, and that the percentage of female candidates awarded II.2s and below was 2.5 times the percentage of male candidates in the same range.

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 5.

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1.1	43	37.23	7.54	68.47	14.64
B1.2	64	33.39	8.8	66.52	12.72
B2.1	36	35.31	8.26	73.89	11.45
B2.2	44	31.14	7.36	71.05	10.26
B3.1	46	35.72	8.96	74.5	13.08
B3.2	15	39.8	8.85	78.67	15.7
B3.3	21	39	8.2	75.9	14.54
B3.4	41	36.32	8.29	74.2	13.23
B3.5	34	38.79	5.34	71.06	11.96
B4.1	33	29.94	6.92	68.42	11.14
B4.2	29	32.9	7.4	70.17	10.43
B4.3	2	38.5	2.12	77	4.24
B5.1	47	31.7	6.41	64.32	9.17
B5.2	60	32.67	8.44	65.02	12.16
B5.3	37	28.68	7.18	66.89	9.89
B5.4	34	30.59	8.08	66.12	10.26
B5.5	42	33	7.4	63.24	10.4
B5.6	40	31.52	7.15	66.85	10
B6.1	31	33.06	9.6	64.03	12.34
B6.2	19	35.79	9.02	62.26	15.29
B6.3	18	23.78	8.34	64.44	13.93
B7.1	20	34.25	6.88	66.15	10.24
B7.2	11	28.82	5.44	68.73	7.46
B7.3	23	31.65	6.01	65.43	8.53
B8.1	43	34.49	8.55	69.28	13.73
B8.2	27	36.11	10.38	72.52	16.47
B8.3	58	31.4	7.4	63.52	7.2
B8.4	14	34.71	5.53	65.71	6.12
B8.5	34	34.62	6.02	65.65	8.4
SB1	10	31.8	10.04	65.8	7.98
SB2a	15	27.33	6.89	64.27	10.24
SB2b	26	32.92	8.79	64.96	9.24
SB3a	76	32.54	5.92	67.64	8.79
SB3b	14	28.07	6.94	64.71	6.41
SB4a	22	33.82	7.25	64.73	7.63
SB4b	9	34.11	5.33	66.22	6.48
SB2b-old	1	-	-	-	-
CS3a	5	-	-	-	-
CS4b	8	33.5	6.87	67	13.73
BO1.1	-	-	-	-	
BO1.1X	-	-	-	-	
BN1.1	-	-	-	-	
BN1.2	-	-	-	-	
BEE	-	-	-	-	
BSP	-	337.43	46.32	67.29	9.09
102		-	-	-	-
127		-	-	-	-
129		-	-	-	-

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.60	18.60	4.30	38	0
Q2	18.03	19.06	5.11	29	2
Q3	17.85	17.94	5.33	19	1

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16.8	6.56	40	3
Q2	16.41	16.84	4.51	39	2
Q3	16.38	16.48	4.54	49	1

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.58	16.58	4.39	34	0
Q2	18.6	18.6	5.1	35	0
Q3	16.5	18.66	6.55	3	1

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.13	15.27	5.74	36	1
Q2	16.64	16.53	3.30	41	1
Q3	11.86	12.90	3.68	11	4

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.51	17.86	6.59	30	3
Q2	18.27	18.27	5.79	40	0
Q3	16.52	17.09	5.13	22	1

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.46	21.46	5.85	15	0
Q2	11.42	17	8.65	4	3
Q3	18.81	18.81	5.86	11	0

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.26	15.14	4.44	14	1
Q2	20.75	20.75	3.99	20	0
Q3	24	24	1.30	8	0

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.69	19.69	2.89	36	0
Q2	17.86	18.42	6.33	21	1
Q3	14.88	15.72	6.43	25	2

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.36	19.12	6.02	8	3
Q2	20.23	20.13	2.43	29	1
Q3	18.77	18.77	4.34	31	0

Paper B4.1: Functional Analysis I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.47	12.47	3.25	19	0
Q2	14.96	15.33	4.97	30	1
Q3	15.57	17.11	5.65	17	2

Paper B4.2: Functional Analysis II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16.29	4.53	27	1
Q2	16.80	16.80	4.20	21	0
Q3	14.81	16.1	6.32	10	1

Paper B4.3: Distribution Theory and Fourier Analysis: An Introduction

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19	19	2.82	2	0
Q2	19.5	19.5	0.70	2	0
Q3	-	-	-	-	-

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.72	17.72	4.03	47	0
Q2	12.84	12.84	2.79	13	0
Q3	14.41	14.41	4.37	34	0

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.37	17.37	6.03	29	3
Q2	15.04	15.82	4.66	52	2
Q3	16.05	16.23	5.79	39	1

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.22	18.22	5.00	22	0
Q2	12.85	12.85	3.83	35	0
Q3	11.94	12.35	4.97	17	1

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.23	14.23	4.61	30	0
Q2	15.94	17.76	6.16	17	2
Q3	14.80	14.80	4.98	21	0

Paper B5.5: Further Mathematical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.91	18.82	6.06	34	2
Q2	14.67	14.67	3.34	34	0
Q3	15.43	15.43	3.98	16	0

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.57	13.24	4.58	29	4
Q2	18.05	18.05	4.97	37	0
Q3	12.72	14.92	5.97	14	4

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15	4.50	31	0
Q2	17.96	18.5	6.96	30	1
Q3	5.25	5	1.25	1	3

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.11	17.11	4.41	17	0
Q2	18.23	18.23	4.69	17	0
Q3	14.5	19.75	10.59	4	2

Paper B6.3: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	8.88	9.23	5.7	17	1
Q2	13.25	15.4	6.57	10	2
Q3	10.63	13	6.23	9	2

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12	12.35	3.20	14	2
Q2	17.64	17.64	4.06	14	0
Q3	22.08	22.08	1.62	12	0

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.14	12.5	2.03	6	1
Q2	11.62	13.14	5.60	7	1
Q3	16.66	16.66	3.24	9	0

Paper B7.3: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.66	16.68	4.80	19	2
Q2	17.14	19.66	7.55	6	1
Q3	13.90	13.95	3.82	21	1

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.65	17.65	4.36	38	0
Q2	17.09	17.09	4.89	42	0
Q3	15.66	15.66	5.81	6	0

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.57	17.57	4.97	26	0
Q2	16.87	16.87	7.18	8	0
Q3	16.44	19.15	7.32	20	5

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.22	17.28	3.56	52	1
Q2	12.4	13.27	5.55	18	2
Q3	14.5	14.84	4.53	46	2

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.23	18.23	2.08	13	0
Q2	9.12	12.2	5.27	5	3
Q3	18.8	18.8	2.89	10	0

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.70	17.70	2.83	34	0
Q2	16.91	16.91	3.76	34	0
Q3	3	-	-	0	1

Paper SB1: Applied Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.4	14.4	3.20	5	0
Q2	11.6	11.6	3.43	5	0
Q3	10.33	10.33	0.57	3	0
Q4	15	15	7.07	2	0

Paper SB2a: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.23	13.23	3.83	13	0
Q2	13.53	13.53	4.51	15	0
Q3	17.5	17.5	2.12	2	0

Paper SB2b: Statistical Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.86	13.70	7.80	17	5
Q2	18.04	18.04	4.52	21	0
Q3	16.46	17.42	4.88	14	1

Paper SB3a: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.19	13.79	4.50	54	3
Q2	15.90	16.86	5.56	30	2
Q3	17.81	17.97	3.23	68	1

Paper SB3b: Statistical Lifetime-Models

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.27	15.1	5.60	10	1
Q2	10.85	11.5	4.09	6	1
Q3	14.41	14.41	3.75	12	0

Paper SB4a: Actuarial Science I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.86	17.86	4.25	22	0
Q2	16.23	16.23	4.52	21	0
Q3	8.5	10	2.12	1	1

Paper SB4b: Actuarial Science II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.66	18.66	1.63	6	0
Q2	14.66	14.66	3.61	6	0
Q3	15.87	17.83	4.82	6	2

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above have also been removed.

B1.1: Logic

Problem 1. Part (a) i was generally done without problems. Part ii was mostly well done, though quite a few people gave inadequate proofs. Part (b) was mostly well done. Part (c) was also generally well done, though quite a few people made unnecessarily hard work of i, and a few struggled with how to approach ii.

Problem 2. Part (a) caused very few problems. Part (b) was mostly well done bookwork. Part (c) was found more challenging, and not that many students managed a correct solution to iii, with several incorrectly offering the field of real numbers as a candidate.

Problem 3. Part (a) was generally done without problems. Part (b) i, ii, and ii were also generally well done, though some were careless in iii that one needed to exclude distinct elements in common classes. The compactness argument in Part (c) i was found challenging by some, while part ii was generally well done.

B1.2: Set Theory

Problem 1. Part (a) was generally well done. Part (b) i was often approached in a confused way, iterating power set rather than taking the set of power sets of elements. Parts ii was mostly well done, and also ii though a number of people failed to see how to use the previous parts. Part (c) was generally approached in the right direction, but may people did not find the correct set to apply foundation, or confused the relationships between ordered pairs and their elements, resulting in oversimplifications.

Problem 2. Part (a) was mostly well done, though many oversimplified (or omitted) giving a lower bound in (iv), and a few people confused the base and exponent of the sets in (ii) and (iii). Part (b) was pure bookwork and generally well done. Part (c) i was well done, while ii is subtle and only a few people managed a correct solution, which involved a careful use of replacement.

Problem 3. Part (a) was bookwork and generally well done. Part (b) i was mostly well done, some overly circuitous proofs by induction were given. Part is most naturally done by transfinite induction and was well done. Part iii was more difficult. Many proved correctly that an ordinal of the given form is always limit, by induction. A nice proof offered by a student was to use the interpretation of ordinal product in terms of reverse lexicographic ordering. The other direction was more difficult and only a few correct solutions were given. Part (c) i was bookwork and well done, and part ii was also generally well done, though a few missed the need for an argument about why the maximal map had to be total.

B2.1: Introduction to Representation Theory

Question 1: (a) is standard bookwork. A couple of students confused the definitions of “semisimplicity” and “complete reducibility” which would make (a)(iii) true by definition.

(b): these are similar examples to bookwork/problem sheets. For (b)(iii) we were looking for a justification beyond just stating that this follows from Maschke’s Theorem. If Maschke’s Theorem was proved later in (c)(ii) in full generality, then it was acceptable to just quote it in (b)(iii).

(c) (ii) The vast majority of candidates reproduced the textbook proof of Maschke’s Theorem. This was acceptable, but time consumming, the original intention was for the candidates to apply (c)(i) in order to get an easy proof of (ii).

(d) A common mistake was to restrict the representation to the centre and quote the result for irreducible representations of abelian groups. This is insufficient since the restriction is reducible in general.

Question 2: (a) and (b) were standard and didn’t pose great difficulties. For (b) it was expected that the students provide sufficient evidence that they understand each step involved in the production of the character table.

(c): this has an economical solution if one uses the formula for the character of the induced representation in terms of conjugacy classes (rather than a sum over all the elements of the group).

(d): the hint may have made this part easier than intended.

Question 3: Very few candidates attempted this question. For part (a) it was acceptable to use without proof that $|C|\chi(g)/\chi(e)$ is an algebraic integer. For part (b), it was easy to realise that in every one-dimensional representation α and β could take one of p values each, hence p^2 possibilities. But this is not sufficient, one needs to use an argument involving the abelianisation of H for example to argue that all of these choices are group homomorphisms.

Part (c) involved standard work with representations. For (d), a counting argument gives the result.

B2.2: Commutative Algebra

Q1: Very popular but many candidates struggled with example in (c) missing very simple examples e.g. $Q[t_2.., t_n..]$ and its field of fractions. Surprisingly, many incomplete arguments in (d) using the correspondence $I \longrightarrow Y^{-1}I$ which is not injective for ideals $InY = \emptyset$ which are not prime.

Q2: Very popular question. In (c) few people managed to show rigorously that $tr.deg_f\, ^R/ < f >$ is $n - 1$.

Q3: Few successful attempts. A common mistake was to try to define g by $g(ai) = (mi)$ for randomly chosen $mi\varepsilon f^{-1}(ai)$, but then there is no guarantee that g extend to a homomorphism $g : I \longrightarrow M$.

B3.1: Galois Theory

Surprisingly, a lot of candidates found difficulties on 1(c).

Most of the candidates seem to have correctly understood how to apply the fundamental theorem even to non-standard base fields such as the ones in ex 2 and ex 3.

B3.2: Geometry of Surfaces

All candidates attempted Q1, and most candidates then picked Q3.

In Q1, candidates were sometimes not sufficiently careful in adding up the contributions of the various external angles for the three regions.

In Q2, some candidates did not spot that by linearity the two maps are explicitly determined. So it only remained to write out the formula using linearity properties.

In Q3, candidates sometimes did not spot the exponential map in part (b), or did not mention why the transition maps were holomorphic, or did not pick a radius of a disc dependent on the centre in (b)(ii). Some candidates missed in (c)(ii) that it sufficed to consider the Weierstrass P-function.

B3.3 Algebraic Curves

Question 1: A question with a wide spread of marks, part (e) found difficult by most. Some candidates started by quoting the classification of conics up to projective transformations as $x2 + y2 + z2 = 0 or x2 + y2 = 0 or x2 = 0$, which was not what I was looking for in (a); in (d) this led to $C = D$, which is false in general.

Question 2: Perhaps too easy, for those that knew the bookwork well (nearly everyone), and attempted by the large majority of candidates. I marked it fairly strictly. Common minor mistakes were not to distinguish between p in C3 0 and [p] in CP2 (e.g. to write p in C with $dP/dx(p) = 0$), and not to justify C,L no common component before applying Bzouts Theorem.

Question 3: Attempted by a minority of candidates, but these tended to score highly.

B3.4: Algebraic Number Theory

1. (a), (b) Most students who attempted these did well.

(c) Many students seemed to understand the idea, but struggled with numerical errors in computing the discriminant. It seems more emphasis should have been given in the course to the computation of the norms of the form $N(r - \alpha)$, where r is rational.

(d) The first part about the norm of units was found easy by most students. However, the problem of finding infinitely many units was solved correctly by only a few students.

2. (a) The statements were described correctly by most students who

attempted this part.

(b) Quite a few students solved this correctly, but quite a few also made errors. For example, not a few thought that the P^i were comprime to I. Also, proofs tended to be long-winded. In retrospect, this part may have been too long.

(c), (d) These parts were on the whole challenging. However, the students who had a firm understanding of part (b) did well on them.

(e) This problem was done at least in part by many students, but there were also many small errors leading to wrong conclusions in one of the primes.

3. (a)At least half of this problem is a straightforward application of the Minkowski bound and Dedekind's theorem. However, a minority of students found the precisely correct structure of the class group. It seems looking for the right relation between the primes was rather challenging in examination conditions.

(b)This problem was reasonably easy for students who attempted it, although small errors of argument were not uncommon. For example, some students omitted to remark why the ideal whose cube is $y + \sqrt{-65}$ should be principle. A few students who gave up on part (a) gave up on this argument as well.

(c) This was probably the hardest problem on the examination, and possibly more hints should have been given. Surprisingly few students realised that an element of order 2 was easy to arrange, thereby enabling them to concentrate on order 3. Some students seemed to have remembered the example $\mathbb{Q}[\sqrt{-29}]$.

B3.5 Topology and Groups

Question 1: 15 attempts; average mark 16.1/25

The bookwork on the simplicial approximation theorem clearly put some people off, but those who attempted it did well on this part. Part (b), on finding explicit simplicial maps, was quite straightforward and was generally very well done. Many people found part (c) quite challenging. The key idea is to use the fundamental group. Any subdivision of a finite simplicial complex is finite, and there are only finitely many simplicial maps between two finite simplicial complexes. However, there are infinitely many based homotopy classes of maps from the circle to itself, that are distinguished by their induced homomorphisms between fundamental groups. Hence, there are always some homotopy classes of maps from a simplicial circle K′ to a simplicial circle L that cannot be made simplicial.

Question 2: 49 attempts; average mark 20.2/25

This was a very standard question on material that is central to the course: the Seifert van-Kampen theorem. I was very pleased that so many candidates could answer it well. There were only two main causes of difficulties. Many candidates were careless about basepoints: they frequently placed them at different points within the space at different stages of the argument. And many candidates found part (c) (v) challenging. The space Z that is constructed is homeomorphic to the 2-sphere minus 4 points, which is homeomorphic to the plane minus 3 points, and there is a homotopy retract from this space onto a wedge of 3 circles.

Question 3: 48 attempts; average mark 18.6/25

This question started with the fundamental groups of trees; this part was universally well done. The basic theory of covering spaces in part (b) was also well done. However, part (c) was more challenging. Many candidates gave incorrect constructions of the covering space. The earlier parts of the question were meant to be a guide here. The correct covering space is equal to the space Y defined in (a)(ii) with trees attached to each of its vertices. By the work done in part (a), the elements x and yxy^{-1} do therefore freely generate this subgroup. The final question (c)(iii) was more approached by a direct algebraic argument, which was generally done well.

B4.1: Functional Analysis I

Question 1 (Lipschitz functions)

Many candidates found part (a) challenging. This demanded recall of ideas from Prelims Analysis, with (iii) intended to be hard. All answered (i) easily enough. Subpart (ii) should have been straightforward too but those—a significant number—who appeared to have forgotten the Mean Value Theorem generally struggled, and some long and fallacious arguments were offered (though two answers did provide a correct argument based on the Fundamental Theorem of Calculus instead of the MVT).

Part (b) (from a problem sheet) caused no problems, though it was disappointing that candidates checked the norm properties without explicitly noting that X is closed under the vector space operations.

As anticpated, (c)(i) was found tricky. Many candidates failed to address the three separate the cases involved: both, one or neither of s, t non-zero. Most were unable to add and subtract terms so as to get an estimate of the right form in the case that s and t are non-zero.

Question 2 (shift operator on ℓ^∞; Hahn–Banach Theorem)

This question was answered by almost all candidates and marks were well spread.

The first part of (a) was standard but not all candidates mentioned that the operator was linear. The second part discriminated well. Most confirmed that the distance from $\mathbf{1}$ to M is at most 1. Tho reverse inequality needs a proof by contradiction. Those who realised this and looked at individual coordinates understood how the argument should go.

Part (b) was a variant on bookwork, similar to a standard HBT application concerning the separation of a point from a *closed* subspace. By no means all candidates appreciated why the fact that $\mathrm{dist}(\mathbf{1}, M)$ was known to be non-zero made closedness of M irrelevant. Various invalid, and often laborious, attempts to prove that M is closed were supplied.

Part (c) was not found difficult by those who looked at powers of T, though the handling of limits sometimes lacked precision. There was a cheap mark to be had in (d) for noting that the shift operator in (a) was still bounded and linear when the scalar field was taken to be $\mathbb{C}$ but only a few saw how to revise the answer to the second part of (a).

Question 3 (dual spaces; density)

The bookwork in (b) was handled well, and the theoretical applications of it in (c)(i) and (d) (largely seen before) caused few problems. Candidates who worked with finite-dimensional spaces to give the example sought in (c)(i) handled this well. Otherwise, almost all attempts overlooked the fact that a subspace must carry the inherited norm.

Most candidates realised that in (c)(ii) the first example hinged on density and the third on separability. In (β) the norm on X_2 was not made explicit and any meaningful interpretation was accepted; however some candidates claimed that a space and a proper closed subspace of it could not have the same dual space up to isometric isomorphism.

Candidates who did not draw a commutative diagram for the last part of (d) were liable to get confused. Here, and in (c)(i) too, candidates tended not to note explicitly that the codomain space was a Banach space—without this their applications of (b) would not have been legitimate.

B4.2: Functional Analysis II

Q1: This question was attempted by almost all candidates. The bookwork parts of the question were handled mostly well with some minor exceptions. The second half of (a)(iii) caused some trouble, though many candidates had a feeling that it was related to the closedness of the range. A number of candidates used the projection theorem for part (b)(ii), but this complicated the matter. Part (b)(iii) appeared to be the most challenging part of the question where one had a problem showing $AA * A = A$ and most candidates did not show uniqueness.

Q2: This question contained partly newly introduced materials in 2017-2018 and was attempted by about two thirds of the candidates. The bookwork parts of the question were handled mostly well with some minor exceptions. Part (c) is an application of the principle of uniform boundedness and those who tried this part typically produced good answers. Quite a number of candidates had a problem with parts (a)(ii) and (iii). Typical answers either used facts about orthonormal sequences or appealed to materials from Part A Integration.

Q3: This question was tried by about one third of the candidates. Most of the question was answered reasonably well when tried, except for (a)(ii) - although a similar example was seen in the lecture. It involved two consecutive applications of the closed graph theorem, but most candidates stopped within or right after his/her first application of the theorem.

B4.3: Distribution Theory and Fourier Analysis: An Introduction

The exam went well and the candidates had a clear preference for Q1 and 2. The marks lost were apparently mainly due to lack of time (explicitly mentioned in one script)

B5.1: Stochastic Modelling and Biological Processes

Question 1. This question was answered by all candidates. Parts (a), (b) and (c) were all well answered, except for the final part of (b). In part (d), most candidates could derive the partial differential equation, however the majority struggled when it came to solving it.

Question 2. This question was answered by relatively few candidates. Parts (a) and (b) are standard and were well done by the majority. Candidates struggled on both parts (c) and (d).

Question 3. This question was answered by most of the candidates. Part (a) is bookwork and was well done. In (b) many students failed to see the relevant transformation in order to use the hint. In (c) many candidates failed to correctly calculate the mean squared displacement. Part (d) was relatively well answered.

B5.2: Applied PDEs

Q1. Part a was generally well done. Full marks in part b required careful attention to detail in formulating Green's function, including signs of outward normal derivatives and the need for a finite domain to apply Green's Theorem. Most candidates recognised the need for method of images in b ii), but there were a number of mistakes in obtaining the needed 7 image points with proper signs.

Q2. This question was attempted by most candidates. There were various valid approaches in part b, but all required determining (p_0, q_0) at the given point on the boundary from the slope at that point. Some sloppy errors occurred in part c plotting rays that did not meet the envelope curve tangentially.

Q3. The first part was done very well by most candidates. The Riemann derivation in b was effectively standard, even though the boundary was not a single smooth curve. The most common mistake was in not seeing the necessary shift $x - \xi$, $t - \tau$ and conversion of boundary conditions to $R = 1$ on $x = \xi, t = \tau$ in order to relate R to the previous similarity solutions.

B5.3: Viscous Flow

The first question was really an exercise in index notation, and easy for those in control of this; but there was plenty of scope for error.

The second question on flow in a wedge had the only genuinely difficult part of the exam, finding inequalities for an integral, and was attempted by almost everyone, with only moderate success; the high score was 22 (from the 46 score), there were two 20s, and the rest below this; there were four lows of 8. It was surprising how few students (those properly taught in college I suppose) knew how to integrate the non-linear oscillator equation $x + V(x) = 0$.

The third question on lubrication theory illuminated the difficulties students had with non-dimensionalisation, surprisingly in my view. The high on this was 21, but it was generally poorly done, with most scripts showing a failure to comprehend the basic scaling and term balance of the theory. It may be of note that fall-off in lecture attendance meant most students would not have seen this material in lecture.

B5.4: Waves and Compressible Flow

Q1 The book work in part (a) was very well done though a handful of candidates carelessly lost a mark for failing to justify the relevant linearised version of Bernoulli's equation. Part (b) was also well done with all but a handful of candidates correctly using the boundedness of $\phi(0, t)$ to deduce that $f(0) = 0$. Almost all candidates handled efficiently the bookwork in part (c)(i) and good progress was made by about half of the candidates on part (c)(ii), though many solutions where inefficient. In part (c)(iii) there were many good attempts at the first part, but only a handful of candidates made substantial progress with the tail.

Q2 Unfortunately there were two identical typos in the bookwork in Q2(a): the two integrals should have ended with $\mathrm{d}k$ rather than $\mathrm{d}t$. The correction was announced within 15 minutes of the start of the exam, having been spotted by a candidate. The bookwork in part (a) was very well done and nearly all candidates reproduced in full the derivation that was covered explicitly in lectures and on a problem sheet. Part (b) was similar to an example from the lecture notes and a problem sheet question, but with a significant enough difference that, while good progress was made by the majority of candidates, only a significant minority found the correct dispersion relation $\omega(k) = (B/\rho)^{1/2}|k|^{5/2}$. In part (c) full credit was awarded to all candidates working correctly but with the incorrect dispersion relation, though even those with the correct dispersion relation did not differentiate it correctly except in a handful of cases.

Q3 The book work in part (a)(i) was well done though a significant minority claimed that the conditions conserved mass and energy, rather than mass and momentum. The problem in part (a)(ii) was well handled by all but a handful of candidates either via their own algebraic manipulations or by following the manipulations in a similar example in lectures and on a problem sheet. The book work in part (b)(i) was very well done. The unseen problem in part (b)(ii) was well done by a significant minority of candidates though many other attempts wrote down the correct equations but then failed to combine them to derive the given answer. In part (b)(iii) good progress was made by only a significant minority of candidates despite the problem having been reduced in part (b)(ii) to one very similar to an example from lectures (the dam having constant speed).

B5.5: Further Mathematical Biology

- Question 1 was a variant on standard theory for pattern formation. Most candidates attempted this question and answered it well, completing parts (a) to (d) without difficulty. However, not all were able to explain the two conditions in the stated inequality in part (d). While most candidates attempted part (e), few were able to determine the range of values of k for which spatial patterning is predicted.

- Question 2 was a combination of Law of Mass Action and travelling wave analysis. Few candidates scored high marks on this question, in large part because they were unable to complete part (c), failing to realise that they needed to add the PDEs for u and v in order to derive equation (7). The wording for part (d) was, on reflection, ambiguous but most candidates understood that they were being asked to transform to travelling wave equations and perform a standard phase-plane analysis of the transformed equations. Parts (a), (b) and (d) were well answered, although few candidates stated that they were using the Principle of Mass Balance to derive the governing equations. Additionally, not all candidates realised that the travelling wave has negative wave speed and, as a result, were unable to produce correct phase-plane diagrams. Failure to derive equation (7) meant also that few candidates attempted part (e) and/or realised that when $v = 0$ the dynamics of u were described by the classical Fisher's equation.

- Question 3 was well done, with most candidates being able to complete parts (a), (b) and (c) correctly. Several candidates struggled with part (c), failing to impose the continuity conditions for c when $R_N > 0$. Some candidates struggled with parts (d) and (e), because they did not realise that the structure of the tumour was different when the drug was added (no necrosis and $R_N = 0$ in part (d); with necrosis and $0 < R_N$ in part (e)).

B5.6: Nonlinear Systems

Q1: Most students answered easily Part a (bookwork). The second part was a classification with respect to parameters. Most students fail to do the obvious first step that had been drilled into them since the first lecture (compute the linear eigenvalues) and tried directly to compute a Lyapunov function (which is only necessary when the fixed point is not hyperbolic). Therefore many marks were lost at that step. No student correctly analysed the case $^2 = 1$. Part (c) required a deeper understanding of the material and I was glad to see that some students managed to answer correctly.

Q2: Most students choose Q2 as it involved more direct calculations. Most students answered easily Part a (bookwork). Most students manage to do well and showed good understanding of the underlying material. The main 1 difficulty of the question resided in providing a full analysis of the problem and quite a few students manage to do so, demonstrating a superior understanding of this topic.

Q3: Only a few students attempted this problem. Despite many related exercises, students still struggle with the notion of centre manifold and extended manifold. Only a few managed to obtain the correct dynamics on the extended centre manifold. The last part of the question required a good theoretical understanding of bifurcation theory and a few students did very well.

B6.1: Numerical Solution of Differential Equations I

Very few candidates answered Q3.

Q1: Most students had problems with part (a) and (b), good students solved (c)(i), very few students solved (c)(ii).

Q2: Many students obtained most of the points available.

B6.2: Numerical Solution of Differential Equations II

Question 1 was addressed by most candidates. It was similar to another paper from previous years but the first part (which contained new elements) was not well understood by the majority of candidates.

Question 2 had some subtleties regarding boundary terms and most candidates did not notice that. However, these difficulties did not influence their subsequent confirmation with the remaining sub-questions.

Question 3 was not very popular, but for those who tackled it, it turned out to be very straightforward to follow the suggested steps and they completed it almost entirely.

B6.3: Integer Programming

Q1: Marks ranged from 0-24, most popular question. The mean was 9.75 and the standard deviation 6.29. The students found this problem harder than I expected, many losing points on details to which they did not pay enough attention.

Q2: Marks ranged from 0-23, mean 14.33, standard deviation 6.3. This was the second most popular question. The students found this easier because it contained more familiar material and fewer novel parts.

Q3: Marks ranged from 0-21, mean 11.86, standard deviation 5.79. Candidates had difficulties to show explicitly that knapsack is a special case of (GAP) and several did not know bookwork algorithm for knapsack.

B7.1: Classical Mechanics

Question 1 is on Lagrangian mechanics. Answers to part (a) and (b)(i) were generally of a very high standard. However, very few candidates got anywhere with (b)(ii) (the way to reduce a two-body problem like this to one degree of freedom is covered in the lecture notes). Part (c) could be attempted independently from part (b), and was intended to require a little thought; there were a handful of good answers.

Question 2 is on rigid body mechanics. Parts (a) and (b) are bookwork, and candidates who had learned the course material well scored highly on these parts. Far fewer candidates managed to correctly derive the quadratic equation for $\dot{\varphi}$ in part (c), although many were able to deduce the final inequality from it.

Question 3 is on Hamiltonian mechanics. Part (a) is bookwork, and was answered extremely well. Parts (b) and (c) involve the computation of Poisson brackets. Despite this being a little different from lecture notes/problem sheet questions, every candidate who attempted this question did very well, producing a largely complete solution.

B7.2: Electromagnetism

This section will be published in the Examiners' Report. Please include comments on each of the questions, summarising the standard of answers and noting any common difficulties encountered by the candidates.

Many students seemed to understand the main ideas and basic content of the lecture course however many errors were made when they tried to apply them.

Q1: There were 6 attempts. Students were confused about the appropriate boundary conditions on the surface of the sphere, leading to many erroneous solutions and lost marks particularly in parts (c) and (d).

Q2: There were 7 attempts. Most of the bookwork was well done and students were able to apply the results of part (a) to part (bi). However, there were many mistakes in the calculations in part (bii). Only a couple of students who tried part (c).

Q3: There were 9 attempts. This question had the best answers and there were a few very good attempts.

B7.3 Further Quantum Theory

Q1. This was a fairly standard question that was attempted by most candidates who were by and large able to get respectable marks.

Q2. This was a non standard question but nevertheless on a central part of the course that attracted few attempts. Those that did attempt the question by and large did very well.

Q3. This question again attracted many attempts with good answers to the first part. The second part was less well done, with many not realizing that the standard basis provides obvious energy eigenstates for the free Hamiltonian.

B8.1: Martingales Through Measure Theory

All three questions are quite standard which cover a fair amount of the material in the lectures, even so, most candidates prefer the first two questions, only 7 candidates chose question 3.

Question 1. Part (a) is mainly book-work, and it suggests a different proof of the first Borel-Cantelli lemma, by applying MCT to a trivial series. Most candidates gain the major part. A few candidates missed a mark for not producing an example for Borel-Cantelli lemma without independence assumption. Part (b) cover several important concepts and conclusions involving an independence sequence, together with a standard application of Borel-Cantelli lemma. The answers to this part are all standard, but a few candidates could not argure the convergence by using standard results in Analysis I.

Question 2. The parts (a) to (c) are pretty standard which covered in the lectures, though in a slightly different form. Most candidates however missed marks for not explaining the computations for part (b), and a few candidates had difficulty to prove the identity in (c), rather than using the partition given by the stopping time instead by induction. Many candidates missed about half of the marks for part (e) because don't know the sequence one should apply the strong law.

Question 3. Only few candidates attempt this question, but those who attempted did quite well. Some candidates missed some marks for the last step (b)(iii) for not properly identifing the correct limit.

B8.2: Continuous Martingales and Stochastic Calculus

Overall the quality of solutions to all questions was high.

Question 1: A very popular question. Most marks were lost on part (c), where there was some confusion over how to best express the event $T_a b$??? in terms of countable union.

Question 2: The most popular question. Marks were lost on both parts (c) and (d), but there was not really a consistent trend. Those who attempted this question, typically, performed well on the other question attempted.

Question 3: Very popular with most marks lost on the details of justification of limits in part (b).

B8.3: Mathematical Models of Financial Derivatives

Two typographical errors - in Q2 (d) 2nd bullet point K, should have been R, in Q3(b) $1/26^2 m(m-1)$ should have been $1/26m^2$. Both were quickly spotted and announced.

Q1 was attempted by 73 out of 77 and generally well done. Main problems were distinction between stochastic process S_t and real variable S (and similar).

Q2 was harder than expected; only 24 out of 77 attempted it and only about 18 made a serious attempt.

Q3 was attempted by 64 out of 77 - this tended to be the final question and many answers appeared rushed towards the end.

B8.4: Communication Theory

Question 1 was very popular, and nearly every candidate attempted it. Most candidates got most of 1a, 1b, but few made progress on 1c beyond 1c(i). A common mistake there was to ignore that Y1, Y2 are only independent conditional on X.

Question 2 was less popular and only a handful of candidates attempted it. For 2a, some answers ignored in the definition of divergence the case when q puts zero mass on a point where p puts mass. In 2b, few candidates picked up the hint to first discuss the case of $X = 0, 1$. In

Question 3, nearly all candidates managed to reach 3c, but many candidates had trouble with finishing the calculation of capacity in 3i by simply 1 arguing with input probabilities and instead gave convoluted answers that led to non-explicit expressions for the channel capacity. In 3c(ii), few answers explicitly mentioned that conditional on the second symbol as input, the output is uniform.

B8.5: Graph Theory

Question 1 was generally well done, in particular part (a), where many candidates gave complete, correct, logical answers. In part (b) there tended to be minor slips, and some candidates made logical errors (starting with a tree, not a code, in proving surjectivity). Part (c) is relatively easy if you argue with codes. Many candidates tried to count trees directly, which is possible but harder, and the solutions here were less good.

Question 2 was mostly well done. In part (a) there were surprisingly many small details wrong defining G/e. (b) was mostly well done, though again sometimes with logical errors. (It is clearest to use different notation for the polynomial and the number of colourings.) Part (c) was mostly well done, though the third part proved to be tricky (as intended).

No candidate attempted question 3, which is on material near the end of the course that had not been examined for several years. I had emphasized in lectures that it is examinable!

BO1.1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately. The two carry equal weight when determining a candidate's final mark. The first half of the exam paper (Section A) consists of six extracts from historical mathematical texts, from which candidates must choose two on which to comment; the second half (Section B) gives candidates a choice of three essay topics, from which they must choose one.

Candidates' choices of extracts in Section A were quite evenly spread across the options, with the exception of question 4, which received no answers. Questions $1, 2, 3, 5, 6$ were attempted by $4, 3, 2, 3, 2$ candidates, respectively. In some cases, unfortunately, candidates missed the key point of the extract at hand: for example, in question 2, we see Brouncker's argument for the convergence of an infinite series, but some answers did not mention convergence at all. This extract also features a diagram, which should have been commented upon and interpreted, but candidates mostly focused on the text alone. A point that was similarly missed in question 5 is that Cauchy's language allows us to interpret his definition either as continuity or as uniform continuity, but this ambiguity was not commented upon. Answers in Section A also tended to be quite vague when it came to the significance of the extract in question — candidates asserted the importance of a particular text, but said nothing further to back this up. Moreover, some candidates forgot that the word 'significance' has been emphasised throughout the course as being used in a very particular way: as a shorthand not merely for the 'importance' (if any) of a particular piece of historical mathematics to us now, but also as an indication of its importance at the time of its writing, and as a description of its place within the development of mathematics, even if its impact has been minimal. These candidates' handling of 'significance' was consequently limited.

In Section B, no candidates attempted question 9; three opted for question 7, and four for question 8. Question 7 was the most straightforward of the options here, and was generally done well. Question 8, a harder essay to write, was also quite well done; better marks were obtained by those candidates who acknowledged that the notions of 'space' and 'number' have changed over the centuries, and who thus gave definitions at different stages of their essay.

With regard to the extended coursework essays, most candidates engaged with the topic well — particularly with the material on Grassmann, which is rather difficult. Unfortunately, however, there were few direct quotations from primary sources — these were only discussed in very general terms, with little of the corresponding mathematics being cited in detail. A similar vagueness attended some candidates' discussions of the influence of the figures under consideration — it was asserted that they were 'influential', but nothing else was said to back this up. The better-scoring essays were those that employed materials that went beyond the recommended reading, and that weren't just digests of discussions from classes.

As a stylistic point, the examiners would welcome the use of subheadings in essays, as a way of breaking down the content, and also of indicating a move to a different topic (otherwise, the reading experience may be quite disjointed). Very few of the essays used subheadings, perhaps because of excessive worry over their impact on the wordcount — but this is precisely what the 10% leeway is designed to alleviate. As a further point of style, the examiners noted that some candidates included rather too many footnotes, which distracted from the main content of the essay.

BEE, BSP and BOE essays and projects

Mark reconciliation was handled for essays and projects as part of the same exercise. Some assessors/supervisors did not make the deadline for submitting marks so the procedure was handled on a rolling basis once initial suggested marks were received, but overall the process went smoothly.

If the proposed marks were sufficiently close, as set out in the guidelines, then the supervisor and assessor were informed that the automatic reconciliation procedure would be applied unless they indicated that they wished to discuss the mark further. If the proposed marks differed sufficiently from each other, then the supervisor and assessor were asked to confer in order to agree a mark.

BN1.1: Mathematics Education

The assessment of the course is based on:

- Assignment 1 (Annotated account of a mathematical exploration) 35%
- Assignment 2 (Exploring issues in mathematics education) 35%
- Presentation (On an issue arising from the course) 30%

Each component was double-marked, with Dr Jenni Ingram (JI) plus myself (NA) as assessors. As recorded in the table below, each component was awarded a USM (agreed between assessors for double-marked components), and then an overall USM was allocated according to the weightings above. Where a significant difference between marks awarded by the two assessors arose or marks were across a grade boundary (these are underlined in the table), scripts was discussed in more detail before agreeing a mark.

There were 12 students on the course this yearan increase on last yearand all went on to study for the BN1.2 (Undergraduate Ambassador Scheme) in Hilary Term. We were pleased to be able to award three Firsts and three high Upper Seconds, although this years marks have a slightly lower mean in comparison with the previous year. We continue to be cautious about awarding scores greater than 80 but were reassured by feedback from Examiners last year suggesting that candidates marks for this module were in line with marks being awarded for their other Part B options.

BN1.2: Undergraduate Ambassadors Scheme

The assessment of the course is based on:

- A Journal of Activities (20%)
- The End of Course Report, Calculus Questionnaire and write-up (35%)
- A Presentation (and associated analysis) (30%)
- A Teacher Report (15%)

Each component was double-marked, with Dr Jenni Ingram (JI) plus myself (NA) as assessors. As recorded in the table below, each component was awarded a USM (agreed between assessors for double-marked components), and then an overall USM was allocated according to the weightings above. Where a significant difference between marks awarded by the two assessors arose or marks were across a grade boundary (these are underlined in the table), scripts was discussed in more detail before agreeing a mark.

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Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1: Applied and Computational Statistics

SB2a: Foundations of Statistical Inference

BS2b: Statistical Machine Learning

SB3a: Applied Probability

SB3b: Statistical Lifetime Models

SB4a: Actuarial Science I

SB4b: Actuarial Science II

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

OCS1: Lambda Calculus & Types

OCS2: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

102: Knowledge and Reality

127: Philosophical Logic

129: Early Modern Philosophy

E. Comments on performance of identifiable individuals

Removed from public version.

F. Names of members of the Board of Examiners

- **Examiners:**
  Prof Helen Byrne (Chair)
  Prof Philip Candelas
  Prof Simon Blackburn (External)
  Prof Michal Branicki (External)
  Prof Jan Kristensen
  Prof Frances Kirwan
  Dr. Neil Laws
  Prof Ben Green

- **Assessors:**
  Dr Nick Andrews
  Prof Ruth Baker
  Prof Dmitry Belyaev
  Prof Helen Byrne
  Prof Coralia Cartis
  Prof Jon Chapman
  Prof Dan Ciubotaru
  Prof Andrew Dancer
  Prof Xenia de la Ossa
  Prof Jeffrey Dewynne
  Prof Artur Ekert
  Prof Radek Erban
  Prof Alison Etheridge
  Prof Patrick Farrell
  Prof Victor Flynn
  Prof Andrew Fowler
  Prof Alain Goriely
  Prof Ben Green
  Dr Stephen Haben
  Dr Abdul-Lateef Haji-Ali
  Prof Ben Hambly
  Dr Heather Harrington

Prof Ian Hewitt
Dr Christopher Hollings
Dr Jenni Ingram
Prof Dominic Joyce
Prof Minhyong Kim
Prof Andrew Krause
Prof Yakov Kremnitzer
Prof Jan Kristensen
Prof Marc Lackenby
Prof Alan Lauder
Prof Terry Lyons
Prof Lionel Mason
Prof Kevin McGerty
Dr Giacomo Micheli
Prof Derek Moulton
Prof Nikolay Nikolov
Prof Luc Nguyen
Prof Harald Oberhauser
Prof James Oliver
Dr Alberto Paganini
Prof Jonathan Pila
Prof Hilary Priestley
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Prof Alexander Ritter
Dr Ricardo Ruiz Baier
Prof Alexander Scott
Dr Gregory Seregin
Prof James Sparks
Prof Gabriel Stylianides
Dr Philippe Trinh
Dr Robert Van Gorder
Prof Sarah Waters
Dr Catherine Wilkins